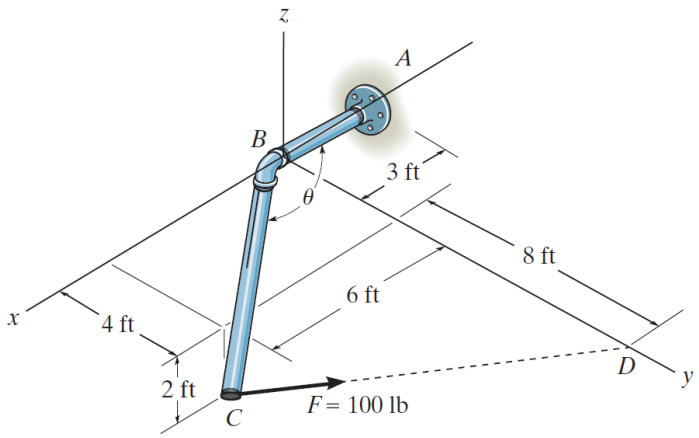
Problem 2-127

Determine the angle θ between pipe segments BA and BC.



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Solution

Write the position vectors to the points A, B, and C.

$$\mathbf{r}_A = \langle -3, 0, 0 \rangle \text{ ft}$$

$$\mathbf{r}_B = \langle 0, 0, 0 \rangle \text{ ft}$$

$$\mathbf{r}_C = \langle 6, 4, -2 \rangle \text{ ft}$$

The unit vector going from B to A is

$$\hat{\mathbf{u}}_{BA} = \frac{\mathbf{r}_A - \mathbf{r}_B}{|\mathbf{r}_A - \mathbf{r}_B|} = \frac{\langle -3, 0, 0 \rangle}{\sqrt{(-3)^2 + (0)^2 + (0)^2}},$$

and the unit vector going from B to C is

$$\hat{\mathbf{u}}_{BC} = \frac{\mathbf{r}_C - \mathbf{r}_B}{|\mathbf{r}_C - \mathbf{r}_B|} = \frac{\langle 6, 4, -2 \rangle}{\sqrt{(6)^2 + (4)^2 + (-2)^2}}.$$

Take the dot product of these unit vectors.

$$\cos\theta = \hat{\mathbf{u}}_{BA} \cdot \hat{\mathbf{u}}_{BC} = \frac{\langle -3, 0, 0 \rangle}{\sqrt{(-3)^2 + (0)^2 + (0)^2}} \cdot \frac{\langle 6, 4, -2 \rangle}{\sqrt{(6)^2 + (4)^2 + (-2)^2}} = -\frac{3}{\sqrt{14}}$$

Therefore, the angle between the pipe segments is

$$\theta = \cos^{-1}\left(-\frac{3}{\sqrt{14}}\right) \approx 143°.$$